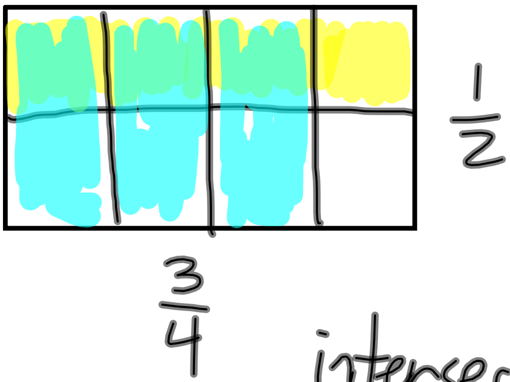
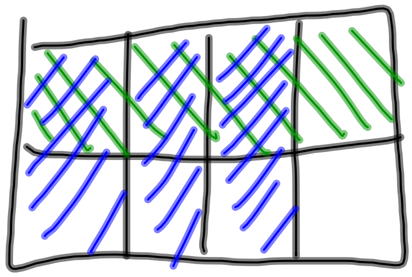
6.3: Multiplication and Division of Rational Numbers

Definition: If $\frac{a}{b}$ and $\frac{c}{d}$ are rational numbers, then $\frac{a}{b} \cdot \frac{c}{d} = \frac{a \cdot c}{b \cdot d}$.

Example: Draw a figure to represent $\frac{1}{2} \cdot \frac{3}{4} = \frac{3}{8}$.





intersection of shading is numerator
number of squares is denominator

Example: Calculate $\frac{27}{62} \cdot \frac{8}{54}$.

$$\frac{\overset{1}{\cancel{27}}}{\underset{31}{\cancel{62}}} \cdot \frac{\overset{\cancel{4}\ 2}{\cancel{8}}}{\underset{\cancel{27}\ 1}{\cancel{54}}} = \boxed{\frac{2}{31}}$$

$$\frac{\overset{1}{\cancel{27}}}{\underset{31}{\cancel{62}}} \cdot \frac{\overset{\cancel{4}\ 2}{\cancel{8}}}{\underset{\cancel{27}\ 1}{\cancel{54}}}$$

1 2 3

Example: Calculate $\frac{18}{44} \cdot \frac{55}{27}$.

$$\frac{\overset{1}{\cancel{\overset{9}{\cancel{18}}}}}{\underset{2}{\cancel{\underset{4}{\cancel{44}}}}} \cdot \frac{\overset{5}{\cancel{55}}}{\underset{3}{\cancel{27}}} = \boxed{\frac{5}{6}}$$

1 2 3

$$\frac{\overset{1}{\cancel{\overset{2}{\cancel{18}}}}}{\underset{2}{\cancel{\underset{4}{\cancel{44}}}}} \cdot \frac{\overset{5}{\cancel{55}}}{\underset{3}{\cancel{27}}} = \boxed{\frac{5}{6}}$$

Fact: The rational numbers over multiplication have the closure, commutative, and associative properites. The following properties also hold.

Identity:

$$1 \cdot \frac{a}{b} = \frac{a}{b} \cdot 1 = \frac{a}{b}$$

Inverse:

$$\frac{a}{b} \cdot \frac{b}{a} = \frac{b}{a} \cdot \frac{a}{b} = 1 \quad \left(\frac{a}{b} \neq 0\right)$$

Zero Multiplication Property: $0 \cdot \frac{a}{b} = \frac{a}{b} \cdot 0 = 0$

Distributive: $\frac{a}{b} \cdot \left(\frac{c}{d} + \frac{e}{f}\right) = \frac{a}{b} \cdot \frac{c}{d} + \frac{a}{b} \cdot \frac{e}{f}$

Example: Calculate the following.

(a) $3\frac{1}{3} \cdot 3\frac{1}{3}$

$$= \Big(\underbrace{3 + \frac{1}{3}}_{\frac{10}{3}}\Big)\Big(\underbrace{3 + \frac{1}{3}}_{\frac{10}{3}}\Big)$$

$$= 3 \cdot 3 + 3 \cdot \frac{1}{3} + \frac{1}{3} \cdot 3 + \frac{1}{3} \cdot \frac{1}{3}$$

$$= 9 + 1 + 1 + \frac{1}{9} = \boxed{11\frac{1}{9}} = \frac{100}{9}$$

(b) $2\frac{2}{3} \cdot 1\frac{1}{4}$

$$\frac{\cancel{8}^{\,2}}{3} \cdot \frac{5}{\cancel{4}_{\,1}} = \frac{10}{3} = \boxed{3\,\frac{1}{3}}$$

6.3 Completed Notes

Definition: If $\frac{a}{b}$ and $\frac{c}{d}$ are rational numbers with $\frac{c}{d} \neq 0$, then $\frac{a}{b} \div \frac{c}{d}$ is the unique rational number $\frac{e}{f}$ such that $\frac{c}{d} \cdot \frac{e}{f} = \frac{a}{b}$.

We will not be studying a model for this in class, but look at p. 390 for some ideas of how to teach this.

Example: Show that $\underbrace{\frac{2}{3} \div \frac{3}{4} = \frac{8}{9}. \text{ because } \frac{3}{4} \times \frac{8}{9} = \frac{2}{3}}$

$$\frac{3}{4} \times \left(\; \right) = \frac{2}{3}$$

$$\frac{\cancel{3}^{1}}{\cancel{4}_{1}} \times \frac{\cancel{8}^{2}}{\cancel{9}_{3}} = \frac{2}{3}$$

6.3 Completed Notes

Example: Show that $\frac{2}{3} \div \frac{3}{4} = n$.

① $\frac{4}{3} \cdot \frac{3}{4} \cdot n = \frac{2}{3} \cdot \frac{4}{3}$

$$n = \frac{2}{3} \cdot \frac{4}{3}$$

② $\dfrac{\frac{2}{3}}{\frac{3}{4}} \cdot \dfrac{\frac{4}{3}}{\frac{4}{3}} = \dfrac{\frac{2}{3} \cdot \frac{4}{3}}{1} = \frac{2}{3} \cdot \frac{4}{3}$

③ $\frac{2}{3} \div \cancel{\frac{3}{4}} \cdot \left(\cancel{\frac{3}{4}} \cdot \frac{4}{3} \right) = \frac{2}{3} \cdot \frac{4}{3}$

Theorem: If $\frac{a}{b}$ and $\frac{c}{d}$ are any rational numbers and $\frac{c}{d} \neq 0$, then $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$. (Keep-Change-Flip)

~~Proof:~~

Example: Compute $\frac{4}{5} \div \frac{12}{5}$ using Keep Change Flip with one of the explanations from before.

$$\frac{4}{5} \div \frac{12}{5} = n$$

Then $\frac{5}{12} \cdot \cancel{\frac{12}{5}}\, n = \frac{4}{5} \cdot \frac{5}{12}$

$$n = \frac{\cancel{4}^{1}}{\cancel{5}_{1}} \cdot \frac{\cancel{5}^{1}}{\cancel{12}_{3}} = \frac{1}{3}$$

$$\frac{3}{4} \div \frac{3}{8}$$

$$= \frac{\frac{3}{4}}{\cancel{\frac{3}{8}}} \cdot \frac{\frac{8}{3}}{\cancel{\frac{8}{3}}} = \frac{\cancel{3}^{1}}{\cancel{4}_{1}} \cdot \frac{\cancel{8}^{2}}{\cancel{3}_{1}} = 2$$